IV. "On Wolf and Rayet's Bright-Line Stars in Cygnus." By WILLIAM HUGGINS, D.C.L., LL.D., F.R.S., and Mrs. HUGGINS. Received November 25, 1890.

In 1867 MM. Wolf and Rayet discovered at the Paris Observatory three small stars in Cygnus, which in the spectroscope showed several bright lines upon a continuous spectrum.\* All three stars have a very bright band in the blue part of the spectrum.

These stars are:-

Their spectra were described in 1873, by Vogel, whose observations agree substantially with the original description given by Wolf and Rayet.† A more complete account of their spectra was given by Vogel in 1883, from observations at Vienna with the 27-inch refractor made by Sir Howard Grubb.‡

Vogel's measures of the bright blue band place it in the star No. 3956 at from  $\lambda$  468 to  $\lambda$  461, with a maximum at  $\lambda$  464; in the star No. 4013 with a maximum at the same place in the spectrum; while the corresponding blue band in the star No. 4001 has a considerably less refrangible position, commencing at  $\lambda$  470, reaching a maximum at  $\lambda$  468, and ending about  $\lambda$  465.

These later measures, though they differ from his earlier ones, in so far as they show that the blue band has not an identical position in all three stars, nevertheless support substantially his earlier observations, which Vogel considered to show, contrary to the statements of Secchi, that the bright lines, including the blue hand, were not due to carbon.

In the diagram, Nos. 1, 2, and 3 show the positions of the bright bands in the three stars, according to Vogel's measures, relatively to the blue band of the hydrocarbon flame.

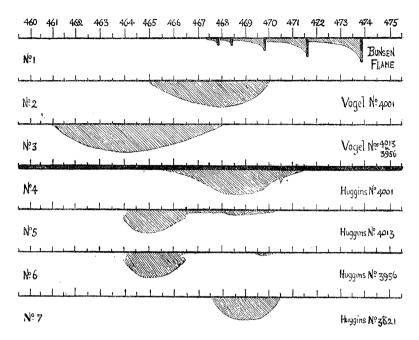
Vogel's measures are:-

	Beginning of	Brightest	End of
	the band.	part.	the band.
Star No. 4001	$\lambda$ 470	λ 468	$\lambda$ 465
,, 4013		$\lambda 464$	1
,, $3956$	$\lambda 468$	$\lambda 464$	$\lambda 461$

<sup>\* &#</sup>x27;Comptes Rendus,' vol. 65, 1867, p. 292.

<sup>† &#</sup>x27;Berichte K. Sächs. Ges. der Wiss.,' Dec., 1873, p. 556.

<sup>† &#</sup>x27;Publicationen Astrophys. Observ. Potsdam,' vol. 4, No. 14, pp. 17—21.



His diagram shows the band in No. 4013 to begin and end at about the same positions as in the star 3956.

It has been stated recently that the bright blue band in all three stars is the carbon band in the blue, commencing near  $\lambda$  474;\* and more recently, notwithstanding the difference of position, according to Vogel, of the band in one of the stars from that which it occupies in the other two of as much as  $\lambda$  0040, that direct comparisons showed

\* Professor Lockyer, in the Bakerian Lecture for 1888 ('Roy. Soc. Proc.,' vol. 44, p. 37), says of the star No. 4001:—"The bright band with its maximum at  $\lambda$  468 is the bright carbon fluting commencing at  $\lambda$  474 and extending towards the blue, with its maximum at 468, as photographed at Kensington."

Of the star 4013:—"The bright band in the blue at 473 is most probably the carbon band bright upon a faint continuous spectrum, this producing the absorption from 486 to 473" (loc. cit., p. 41).

Of the star No. 3956:—"The bright band at 470 is the carbon band in the blue, commencing at 474, with its maximum at about 468, as observed and photographed at Kensington" (loc. cit., p. 43). See Vogel's measures for the band in this star, which are given in the text.

Diagrams of the spectra of these stars are given at pp. 38, 40, and 41, based on Vogel's observations and his curves, which, on a slightly reduced scale, are placed at the bottom of the diagrams. The maximum of Vogel's curves is placed in all three diagrams at  $\lambda$  468, and agrees in the diagrams with the carbon band, whereas Vogel's original curves and his measures place the maximum in the case of two of the stars at  $\lambda$  464, beyond the carbon band.

an absolute coincidence of the band in all three stars with the blue band of a spirit-lamp flame.\*

As the presence or absence of carbon in these stars, as shown by the coincidence or otherwise of the blue band with that of the hydrocarbon flame, was of great importance to us in connexion with a wider investigation on which we are at work, we thought it necessary, after these recent statements as to the position of the band, to make direct comparisons of the spectra of these stars with that of the hydrocarbon flame under sufficiently large dispersion to enable us to determine whether Vogel's measures are substantially correct, or whether they are so largely in error as the absolute coincidence of the band with the blue band of a spirit-lamp flame in the case of all three stars would show them to be.

The obvious importance of making the observations with sufficient dispersion is supported by Vogel's own experience. With the small dispersion which he employed in his earlier observations in 1873, he did not detect the large difference of position, about λ 0040, of the band in No. 4001, as compared with its position in the other two stars. On this point Vogel says, in his memoir of 1883:—"Etwas abweichend ist nur die Auffassung der Lage der breiten hellen Bande im Blau, die bei den früheren Messungen bei allen drei Sternen übereinstimmt. . . . Bei den verhältnissmässig geringen optischen Hülfsmitteln, mit denen jene Messungen ausgeführt wurden, ist die Uebereinstimmung aber eine ganz überraschende" (loc. cit., p. 21).

We observed the spectra of the stars successively, first with a direct vision prism of small dispersion, then with a spectroscope (A) containing one prism of 60°, and finally with a spectroscope (B) with two compound prisms, equal to about four prisms of 60°; with the last-named instrument the comparisons with the hydrocarbon flame were made.

A rapid preliminary comparison in the spectroscope (B) of the spectra of the three stars with the blue base of a Bunsen flame

<sup>\*</sup> Professor Lockyer, in a signed article in 'Nature' (August 7, 1890, vol. 42, p. 344), writes;—

<sup>&</sup>quot;In the Bakerian Lecture for 1888 I gave a complete discussion of the spectra of bright-lined stars, as far as the observations went, and the conclusion arrived at was that they were nothing more than swarms of meteorites a little more condensed than those which we know as nebulæ. The main argument in favour of this conclusion was the presence of the bright fluting of carbon which extends from 468 to 474. This standing out bright beyond their short continuous spectrum gives rise to an apparent absorption band in the blue. . . . Direct comparisons of the spectrum of all the three stars in Cygnus with the flame of a spirit-lamp have been made by Mr. Fowler, and these showed an absolute coincidence of the bright band in the stars with the blue band of carbon seen in the flame. It was found quite easy to get the narrow spectrum of the star superposed upon the broader spectrum of the flame so that both could be observed simultaneously."

showed at once the substantial accuracy of Vogel's measures, and the striking difference of position of the band in the star No. 4001 from that which it holds in the other two stars.

The obvious want of agreement of the star bands with the blue band of the Bunsen flame was seen at once. Their relative positions appeared to agree substantially with the positions represented in No. 2 and No. 3 of the diagram, which are based on Vogel's measures. More careful and repeated observations brought out clearly, as is indeed shown by Vogel's curve, that the star bands differ in character as well as in position from the blue band of the hydrocarbon flame, and also in some respects from each other.

Before giving in more detail the results of our observation on each of the three stars, it should be stated that in all the stars the continuous spectrum is not in our instruments a short one, ending before the position of the bright blue band is reached. On the contrary, an examination with all three spectroscopes showed that the continuous spectrum, though enfeebled by absorption a little before reaching the blue band, can be traced, as is shown in Vogel's curves, quite up to the band, and indeed extends for a long distance into the violet beyond the blue band. The blue band does not in our instruments stand out bright beyond the end of a short continuous spectrum, but falls upon a fairly luminous continuous spectrum, which can be traced past the blue band into the violet, apparently as far as the eye could be expected to follow it.

We suspected bright lines or bands in the region more refrangible than the blue band, but in such faint objects this is a point which should be determined by photography.

Professor E. C. Pickering has since kindly informed us that his photographs of the star No. 4001, which extend into the ultra-violet region, show beyond the blue band the bright hydrogen lines at 434, 410, 397, and 389; and also other bright lines at 462, 455, 420, 406, 402, 395, and 388.

In his photographs of the stars 4013 and 3956, however, the only well-marked line is in the blue at 470.

Star 4001.—In this star, as is shown by Vogel's measures and curve, the bright blue band is less refrangible than in the other two stars, and approaches therefore nearer to the position of the blue band of the hydrocarbon flame. The appearance and position of the band in the star as contrasted with that of carbon, when observed in spectroscope B, are represented in spectrum No. 4 of the diagram.

The brightest part of the band, from about  $\lambda$  468 to  $\lambda$  469, falls off rather suddenly in brightness at about these wave-lengths, but can be traced towards the red as far as about  $\lambda$  471.5, and as far in the blue as about  $\lambda$  465.5.

In our observations of this and the other stars we did not attempt micrometric measures of the blue band, but we estimated their positions by means of the intervals between the five flutings of the band of the Bunsen flame. In the case of objects so faint in our instrument when viewed under the dispersion of spectroscope B, we did not consider there would be any real gain of accuracy by attempting to take measures.

Though the wave-lengths assigned to our positions must therefore be regarded as not more than approximately correct, we have no hesitation in considering them fully accurate enough for the purpose of our investigation.

The star band is not split up into well-separated maxima, as is the Bunsen flame band, but we have little doubt that the brightest part of the band, from  $\lambda$  468 to  $\lambda$  469, which is much, and rather suddenly, brighter than its beginning and termination, consists of bright lines. Lines appear to flash out at moments, but in our instruments they cannot be seen with sufficient steadiness for us to be sure of their number and position.

Under certain conditions of the electric discharge, the normal relative brightness of the component flutings of the blue hydrocarbon band has been observed to be so far changed that the position of maximum intensity is moved from the less refrangible end of the band towards the blue end; but the five flutings remain without any change of their position in the spectrum.\*

Dr. Hasselberg, by means of feeble disruptive discharges from tinfoil terminals placed outside an exhausted tube containing vapour of benzole, obtained a nearly pure spectrum of the order of that in a hydrocarbon flame mixed only with faint lines of hydrogen. He says: "Es war aber hier die violette Gruppe sehr schwach. Dagegen schien mir die blaue Gruppe relativ heller als im Flammenspectrum, und sie hatte ausserdem entschieden ihre grösste Intensität nicht an der weniger brechbaren Kante, sondern mehr nach dem Violetten hin. Dasselbe schien mir auch mit der gelben Gruppe der Fall zu sein. In Bezug auf die grüne Gruppe konnte ich aber keine Verschiebung des Intensitätsmaximums bemerken."

Dr. Hasselberg gives curves to show the amount of this change of intensity in the blue group and in the orange group. In the blue group the maximum is moved from the first to the third line, that

\* "It is necessary to state that the maximum luminosity of the blue band, under some conditions, is about 468. . . . The conditions under which this band has its maximum luminosity at 468 in Geissler tubes seem to be those of maximum conductivity. If the pressure be high, all the members of the group are sharp, and the luminosity of the band is almost uniform throughout. This always occurs when the pressure is very low. At intermediate stages of pressure, however, the luminosity has a very decided maximum at about 468" (Appendix to the Bakerian Lecture for 1888, 'Roy. Soc. Proc.,' vol. 45, pp. 167, 168).

is, to about  $\lambda$  4698. His curve gives the brightness of the maximum over that of the first line as about 7 to 6, whereas the normal relative intensity of these two lines is in the inverse direction and as about 2 to 4 (Watts, 'Index of Spectra,' p. 30).\*

A similar change from the normal relation of brightness of the flutings within the band, even if removed to  $\lambda$  468, does not seem to us to bring the star band sufficiently into accordance in character and position with those of the band of the hydrocarbon flame to justify us in attributing the blue band in the star to carbon. Though we traced the band a little further towards the red, than the position of the beginning of the band given by Vogel's measures, yet it is very faint, and without any increase in brightness at the place of the second fluting of the carbon band, beyond which we were unable to see it.

According to Hasselberg's curve, the second bright fluting, where in our instruments the star band ends, still retains a brightness of about 11/12 of that of the maximum, and the first line, at the position of which no brightening of the feeble continuous spectrum of the star could be detected, a brightness of about 6/7 of that of the maximum. That the flutings of the band were not obscured by the absorption band at this part of the spectrum appears clear from the circumstance that we could trace the faint continuous spectrum up to the bright band.

Vogel's and our observations agree in making the band run on some distance beyond the visible termination of the blue band of the Bunsen flame. Piazzi Smyth, under some conditions, observed a large number of faint "linelets" beyond the "5th leader" of the band, where its visibility usually ends; and in the brilliant light of the arc the band can be traced further in the blue. The extension of the band under such circumstances does not seem to us to affect our present argument; for in the very feeble light of the star we may surely take it that the carbon band, if present, could not be seen to extend further than its usual visible limit in a Bunsen flame, namely, about  $\lambda$  468.

Perhaps it should be stated in connexion with the circumstance that we saw the band extend a little further towards the red than Vogel did, that at the time of our observations the hydrogen line at F was not visible in our instruments, whereas it was bright at the time when Vogel observed the star. In the spectrum of a similar star, D.M. +37° 3821, in which the hydrogen line at F at the time was bright, the blue band was seen by us to stop near the place given by Vogel in his measures of the star No. 4001.

Not only is there no coincidence, so far as Vogel and we have observed, of the position of the band in the star with that of the blue

<sup>\* &#</sup>x27;Mém. de l'Acad. Imp. des Sciences de St. Pétersbourg,' vol. 22, No. 2, 1880, p. 82.

band of the Bunsen flame; but, further, the want of accordance of its general characters is so great as to make the view that its origin is carbon very improbable. This improbability is very greatly increased when we find, as will be shown presently, that no traces whatever of the very bright beginnings of the more brilliant green and orange bands could be detected by us in any of the stars. Further, Professor E. C. Pickering has kindly sent to us an account of his photographs of this star, which, though they show the hydrogen line at  $\lambda$  434, do not exhibit any brightness at the positions of the indigo hydrocarbon bands, beginning near 4312, and  $\lambda$  4382.

This star, however, can scarcely be taken by itself; in the case of the other two stars, in the spectra of which, according to Vogel's, Copeland's, and our own observations, the brightest part of the blue band is from  $\lambda$  464 to  $\lambda$  465, but nearer  $\lambda$  465, quite outside the ordinary visible limit of the carbon band, the evidence seems very strong indeed that the band does not owe its origin to carbon.

We satisfied ourselves that when the spectrum of the star is examined under the dispersion of spectroscope B, none of the brighter parts of its spectrum fell at, or very near, the green, orange, and indigo flutings of the hydrocarbon flame spectrum; at these positions we were unable to detect any sensible brightening of the star's spectrum. Professor Copeland's measure of the blue band in 1884 was  $\lambda$  469.5.

No. 4013.—Vogel does not give measures of the beginning and the ending of the band in this star, but only of the brightest part:— "Hellste Stelle, nahezu Mitte, einer breiten verwaschenen Bande,  $\lambda$  464." He gives, however, a diagram of the spectrum in which the bright blue band is represented as substantially coincident in position and in general character with that in the spectrum of No. 3956.

Our observations agree substantially with those of Vogel, but they make the band to consist of two parts—a very bright part, from about  $\lambda$  466 to  $\lambda$  464, but brightest near  $\lambda$  465, and a very faint band, apparently detached from the bright one from about  $\lambda$  4685 to about  $\lambda$  4705. This faint band is brightest near where it ends rather abruptly at the more refrangible end. The very bright band has not the character of a fluting, nor is it broken up into maxima widely separated like those of the Bunsen flame band, but appears to be a group of bright lines. The lines were only glimpsed at moments; it is therefore difficult to make a drawing which truly represents the character of the band as seen in our instruments. The band, which is shown at No. 5 of the diagram, is left unfinished at the more refrangible end, as we were not certain how far we ought to consider it to extend.

In this star (as we shall show to be the case in No. 3956 also), the

great body of bright radiation lies far beyond the ordinary visible limit of the blue carbon band, and no connexion whatever with carbon is even suggested to us by the star's spectrum. Dr. Copeland's measure of the band in 1884 was  $\lambda$  465.4.

The continuous spectrum of the star is unequally bright from the presence of bright groups and also apparently of absorption bands or lines, and therefore with small dispersion it might be easily supposed that the spectrum is brighter at the position of the green carbon band. We examined the continuous spectrum repeatedly with great care, and we were able to satisfy ourselves that, under the considerable dispersion of our instruments, there was no sensible brightening of the spectrum at the positions of the green and of the orange bands of the Bunsen flame.

No. 3956.—Vogel places the brightest part of the band in this star at the same position in the spectrum as in the star last considered, No. 4013, namely, at  $\lambda$  464, a position beyond the carbon band. The position of the band as it appeared in spectroscope B with the third eye-piece, is represented at No. 6 in the diagram. The position of the band relatively to that of the Bunsen flame was determined by estimations made by means of the intervals between the bright flutings of the Bunsen band. The position agrees substantially with that given by Vogel, but places the maximum brightness nearer to 465. This bright part probably consists of a group of bright lines and falls off rather suddenly at both ends. We were not certain if the light beyond this bright part was due to a continuation of the band or to the continuous spectrum, more or less dimmed by absorption; we have, therefore, left the ends of the band incompleted in the diagram. Copeland's measure of this band in 1884 was  $\lambda$  464.9.

The sub-band seen in the star No. 4013 is very much fainter in this star, but we have little doubt that there is a very faint band present at about the same place in the spectrum.

Professor E. C. Pickering has found in the near neighbourhood of these three stars other stars possessing bright lines in their spectra.\* The brightest of these, independently discovered by Dr. Copeland in 1884,† namely, D.M. +37° 3821, in which the spectrum is similar to that of the Wolf-Rayet stars, was examined. Dr. Copeland says of this star:—"It has a spectrum of several bright lines near D, and a very bright band in wave-length 464" (loc. cit.). We were therefore

<sup>\* &</sup>quot;The following list contains the designations of all eight stars (with bright lines), the first four being those previously known:—35° 4001, 35° 4013, 36° 3956, 36° 3987, 37° 3821, 38° 4010, 37° 3871, 35° 3952 or 3953. Of these 37° 3871 is P Cygni, and 37° 3821 is the star in the spectrum of which the bright lines are most distinct" (letter in 'Nature,' vol. 34, p. 440).

<sup>† &#</sup>x27;Monthly Notices, R.A.S.,' vol. 45, p. 91, 1884.

surprised to find the blue band, which is very brilliant, not in the position of the band in the stars No. 4013 and No. 3956, but less refrangible, corresponding to the position of the band in the star No. 4001.

The bright band begins about  $\lambda$  467 and runs on to nearly  $\lambda$  470.5. It is clearly not made up of flutings similar to those of the Bunsen flame, but is a group of lines nearly uniformly bright throughout the length of the band. The band did not appear to extend in our instruments towards the red quite so far as the band of No. 4001; it stops near the place assigned by Vogel to the beginning of the band of No. 4001.

The band is represented in spectrum No. 7 in the diagram. Direct comparison with hydrogen showed that the line at F is brilliant in this star.

After some scrutiny of this part of the star's spectrum, we became conscious of a very feeble brightening of the spectrum beyond the bright band towards the violet, and as far as we could estimate its position, at about from  $\lambda$  464 to  $\lambda$  467, that is to say, about the position assigned to the band by Dr. Copeland in 1884.

We then re-examined the spectrum of No. 4001, and were able to feel pretty sure that a similar faint brightening of the spectrum occurs in this star also at the same place, namely, about the more refrangible position of the blue band in the stars No. 4013 and No. 3956.

Dr. Copeland, during his travels in the Andes in 1883, observed γ Argûs, and five small stars with bright lines in their spectra. He says:—"As far as my measures and estimates go, all of them belong to the same class as the three Wolf-Rayet stars in the Swan, to which Professor Pickering has since added a fourth outlying member."\*

Dr. Copeland gives the position of the bright blue band in  $\gamma$  Argûs as  $\lambda$  464.6.

Among the stars in the great cluster G.C. 4245, near  $\zeta$  Scorpii, Dr. Copeland found a star, P. XVI 204 = Stone 9168, which has a similar spectrum, namely, with a bright band in the blue and two in the yellow. He found the position of the blue band to be  $\lambda$  465·1.

In the case of two other small stars with similar spectra, he found respectively for the blue band the approximate measures  $\lambda$  463·3 and  $\lambda$  463·6.

These four stars were similar, therefore, at the time of the observations to No. 4013 and No. 3956, in which the maximum of the blue band is not far from  $\lambda$  464, and therefore outside and beyond the ordinary visible limit of the blue carbon band.

<sup>\* &</sup>quot;An Account of some recent Astronomical Experiments at High Elevations in the Andes;" 'Copernicus,' vol. 3, 1883.

Professor Vogel observed two other stars with similar spectra, of which the main feature is the very bright band in the blue region, namely, Arg. Oeltzen 17681 and Lal. 13412. These stars are too low in southern declination to be reached from our observatory.

Vogel places the blue band in Lal. 13412 at  $\lambda$  469, which shows that it has a position similar to that of No. 4001 and of Dr. Copeland's star. In the case of Arg. Oeltzen 17681, Vogel makes the band to extend through about the entire range of refrangibility occupied by the two positions of the blue band in the Wolf-Rayet stars according to his measures of them, namely, from  $\lambda$  461 to  $\lambda$  470, with a maximum at the place where they would overlap, namely,  $\lambda$  466.

Let us consider the four stars with an intensely brilliant blue band which we have examined; in two of them the band extends from about  $\lambda$  464 to  $\lambda$  467, and in the other pair the band has a less refrangible position, from about  $\lambda$  466 to  $\lambda$  471, but there is also in the case of each pair a very faint band visible, or suspected, at the position of the blue band in the other pair. Further, in Arg. Oeltzen 17681, Vogel found the bright band sufficiently long to include both positions of the band.

One suggestion which presents itself is whether these bands, or, more correctly, these groups of bright lines, may be variable, so that, under certain conditions, one or other of them becomes brilliant. Such a state of things would reconcile our observations of +37° 3821 with the earlier measures of Dr. Copeland, and, indeed, might possibly explain, if this variability should be established, the circumstance that so accurate an observer as Professor Vogel did not detect, even with his smaller instrument in 1873, the very large difference of position of the band in 4001 from that of the corresponding band in the stars 4013 and 3956, which was so conspicuous in 1883, and is so still at the present time. In the broad characters of their spectra, and in their magnitudes, the Wolf-Rayet stars have remained unchanged since the discovery of their remarkable spectra in 1867.

As the only direct evidence of such a variability rests upon the change of position of the band in Dr. Copeland's star since his observation of it in 1884, I wrote to Dr. Copeland to ask if his position rested upon sufficiently accurate measures or was arrived at by estimation only. In reply he says:—"The place of the blue line (rather band) in D.M. +37° 3821, given in the 'Monthly Notices,' is a mere estimate to show the character of the star."

Whether any change of position of the band has taken place must therefore remain at present uncertain; but, independently of any such direct evidence of variability, the two positions of the very bright blue band, with the suspicion of faint bands at the alternate positions, appear to us suggestive of possible variation, especially when we consider that the spectra of these stars consist of numerous absorption bands and groups of bright lines upon a feeble continuous spectrum, a character of spectrum which seems to point to a probably unstable condition of the atmospheres of these stars.

The large difference of position of the bands in the two groups of stars is much too great to admit of an explanation founded upon a possible orbital motion of the stars. Besides, the near coincidence of Dr. Copeland's measures of two bright lines common to the stars 4001 and 4013 shows that the difference of position of the blue band is not due to motion in the line of sight.\*

If future observations should show that the bright blue groups are variable, we must look, it would seem, to causes of a physical or a chemical nature.

If the two bright groups, differing in position by about  $\lambda$  0040, belong to different substances, or, less probably, perhaps, to different molecular conditions of the same substance, it is conceivable that one or other substance, or molecular state, may predominate and appear brilliant, according to certain unknown conditions which may prevail in the stars' atmospheres.

It might be suggested that both bands are due to a long group of bright lines, extending from about  $\lambda$  461 to  $\lambda$  471, and that this long group is cut down by absorption bands; in one pair of stars an absorption from the green cuts off the less refrangible part of the long group down to about  $\lambda$  467, while in the other two stars the more refrangible part is eclipsed, and the bright group appears as in 4001.

The appearance of the spectra in our instruments scarcely seems to us to be in accordance with such a view, because, though we did suspect brightenings in the alternate places, the appearance of the spectrum was not such as to suggest a bright group dimmed by absorption, for in that case the amount of absorption needed to all but obliterate a group, as bright as it appears in the other pair of stars, would have blotted out completely the relatively feeble continuous spectrum. This continuous spectrum, though faint, was still distinctly seen.

More observations are needed, but it appeared to us desirable by these suggestions to invite the attention of observers to the points in question.

\* Dr. Copeland permits me to give the following measures of the bright lines in the Wolf-Rayet stars, which were made by him and Mr. Lohse on January 28, 1884.

	1st yellow	2nd yellow	$\mathbf{Bright}$	$\mathbf{Faint}$	Large blue
Star.	line.	line.	line.	line.	band.
+35° 4001			$541 \cdot 2(3)$	522.0(1)	469.5(3)
+ 35° 4013	$582 \cdot 4(2)$	568 · 9 2)	541.0(2)		$465 \cdot 4(2)$
+ 36° 3956	581.0(2)	570 (2)		$523 \cdot 3(1)$	464.9(2)

As the main object of our examination of these stars was to determine whether the bright band in the blue was to be regarded as showing the presence of carbon by its coincidence with the blue band of the hydrocarbon flame, we were not able, from the pressing claims of other work, to extend our examination to many other points in connexion with the spectrum of these faint stars, for an exhaustive examination of which, indeed, our instruments are not sufficiently powerful.

We have stated already that the fairly luminous continuous spectrum reaches up to the bright band in all three stars, and extends beyond into the violet, as far as the eye could be expected to follow it.

The spectra are weakened at many points by what appear to be absorption bands, and are crossed by several brilliant lines, the positions of some of which have been given by Vogel and by Copeland.

An examination with spectroscope B of some of these bright lines, as they appear under small dispersion, showed them to be really not single lines, but short groups of closely-adjacent bright lines.

One of the brightest of these lines is found in the star No. 4013, at the position, according to Vogel, of  $\lambda$  570.

Dr. Copeland's measure for this line is  $\lambda$  568.9 in star 4013, and  $\lambda$  570.4 in the star 3956.

As this position is not very far from that of the green pair of sodium lines at  $\lambda$  5687 and  $\lambda$  5681, it has been suggested that the line in the star is due to sodium, though there is no line of comparable brightness in the star's spectrum at the position of the dominant pair of the sodium spectrum at D.\*

On confronting in spectroscope B the star line with the green sodium lines, the bright space in the star's spectrum was seen to consist of a short group of several bright lines close together, and nearly equally bright. This group appeared to extend through about four times the interval of the sodium pair, which would make the length of the group about  $\lambda$  0024. The green sodium lines cross the group at about one-fourth to one-third of the length of the group from its more refrangible end. The group in the star is rather less bright at the two ends, but there is no gradual shading off in either direction, as in the case of a fluting.

When we examined this part of the spectrum with the small dispersion of a prism of  $45^{\circ}$ , we were pretty sure of a feeble bright line, less refrangible than the pair of bright groups in the yellow, and not far from the position of D. We were not able to see this line in spectroscope B with sufficient clearness to enable us to fix its position. It may be D, or, perhaps more probably  $D_3$ .

<sup>\*</sup> The 570 line is most probably the green sodium line 569, the absence of the yellow sodium being explained by the half-and-half absorption and radiation mentioned in the discussion of the causes which mask and prevent the appearance of a line in a spectrum (Bakerian Lecture for 1888, 'Roy. Soc. Proc.,' vol. 44, p. 41).

In No. 4001, Vogel saw a line at the position of the F line of hydrogen. It is probable that this line, as is the case in so many stars in which it appears bright, is variable, as we were not able to see it when the H $\beta$  line from a vacuum-tube was thrown in. In the similar star D.M. +37° 3821, as we have stated already, the F line of hydrogen was very bright.

We were unable to detect in any of the stars a brightening of the spectrum at the position of the chief line of the bright-line nebulæ. For this examination the lead line at  $\lambda 5004.5$  was thrown in, and the continuous spectrum of the star near to this position carefully scrutinised.

In their original paper, Wolf and Rayet state that they were not able to detect any nebulosity about the stars. They say: "Elles ne présentent non plus aucune trace de nébulosité" (loc. cit., p. 292).

In a recent paper, Mr. Keeler, of the Lick Observatory, confirms this view. He says: "At my request, Mr. Burnham and Mr. Barnard examined the Wolf-Rayet stars in Cygnus for traces of surrounding nebulosity, but with only negative result."\*

Notwithstanding these negative results, it appeared to us of great interest to ascertain further if any nebulosity would come out in a photograph of the stars taken with a long exposure.

Mr. Roberts responded at once to our wish when we asked his invaluable assistance, and on November 1st, of this year, he took a photograph of this region of Cygnus, with an exposure of two hours.

The three stars come out strongly upon the plate, but there is no nebulosity to be seen near any of them. There are faint stars in close proximity to the three stars, and apparently surrounding them, and, in the case of No. 3956, six of these faint stars are seen close to it, in an apparent spiral arrangement.

Though this surrounding of faint stars should be pointed out, it should, at the same time, be stated that the whole neighbouring region is so densely studded with similar faint stars that it would be rash, perhaps, at present to suggest that this apparent connexion of the bright-line stars with faint ones near them may be other than accidental.†

- \* 'Publications of the Astronom. Soc. of the Pacific,' No. 11.
- † [Mr. Roberts has furnished us with the following description of the stars as they appear on his photograph:—
- "No. 4001 appears as a multiple star made up of one bright, two fainter, and one very faint star partly behind the others; there is also a fourth bright star close to the multiple star. The group is surrounded by at least eight faint stars within a radial distance of  $\pm 86$ " of arc from centre to centre.
- "No. 4013.—The photo-image of this star is made up of three stellar images touching each other in a line slightly curved. Two are bright and one faint; and there are indications of two other faint stars behind the two bright ones. This

Professor E. C. Pickering informs me "that photographs have been obtained at the Harvard College Observatory of all the stars hitherto discovered whose spectra consist mainly of bright lines and are of the class discovered by Rayet. Part of these have been photographed at Cambridge, and the remainder in Peru." He states that they may be divided into three sub-classes, according to the characters of their fifteen bright lines. He says, further: "Photographs of the spectrum of planetary nebulæ have also been obtained. They resemble closely the spectra described above, except that the line 500 is strongly marked; 470 is seen in most of them, while the lines due to hydrogen are also bright."

It would seem that Professor Pickering's photographs do not permit him to distinguish the different positions of the bright blue band in some of these stars, for he gives for all the stars the same position, namely,  $\lambda\,470$ .

We regret that the insufficiency of our instrumental means has left our examination of the spectra of these stars less complete than we could wish. Our observations appear to us, however, to be conclusive on the main object of our enquiry, namely, that the bright blue band in the three Wolf-Rayet stars in Cygnus, and in D.M. +37° 3821, is not coincident with the blue band of the Bunsen flame.

V. "On Stokes's Current Function." By R. A. Sampson, Fellow of St. John's College, Cambridge. Communicated by Professor Greenhill, F.R.S. Received November 24, 1890.

## (Abstract.)

In Maxwell's 'Electricity and Magnetism,'\* a view is put forward, in accordance with which we may regard any irrotational motion in a perfect liquid, for which the velocity potential is a solid zonal harmonic, as due to the juxtaposition at the origin, and upon the axis of symmetry, of sinks and sources.

But, in a liquid, any irrotational motion which is symmetrical with respect to an axis gives a velocity potential which may be expressed as a sum of a series of solid zonal harmonics, their common axis being the axis of symmetry, and their origin arbitrary, provided multiple image of four or five stars is surrounded by five bright and seven faint

multiple image of four or five stars is surrounded by five bright and seven faint stars; all within a radial distance of 82" of arc measured from centre to centre of the multiple star. The multiple image measures  $\pm 55$ " in length and  $\pm 19$ " in breadth.

<sup>&</sup>quot;No. 3956.—Its photo-image is  $\pm 27$ " in diameter. It is encircled by three stars of lesser brightness, and six faint ones within a radial distance of 59", i.e., there are nine stars within a radial distance of 59"."—Dec. 5.]

<sup>\*</sup> Vol. 1, chapter ix.

